Bootstrapped Durbin– Watson Test of Autocorrelation for Small Samples

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ABSTRACT

The Durbin-Watson (DW) test is the most widely used test for autocorrelation of a first order in regression analysis. The critical value of DW test depends on X matrix. As a result, the DW test statistic falls sometime in the inconclusive region. For large sample, the DW test can be used for normal distribution. In this paper, we proposed a bootstrap critical value for small sample and compared the power properties with other procedures. Monte-Carlo study shows that the bootstrapped DW test performs better than the usual DW test with the help of power.

INTRODUCTION

In time series or econometric analysis, our aim is to forecast or predict future behavior of variables. It is to estimate the unknown parameters of the model for making the predictions of the natures of variables. But the estimates of the parameters may be inefficient if the error term of the model does not follow certain assumptions. The assumption of the model may not justify in all cases. Usually, autocorrelation problem arises in time series data. If we used in usual testing procedure despite autocorrelation whatever conclusion may be mislead. Thus, it is necessary to detect the existence of autocorrelation in a given series at the initial stage.

The Durbin-Watson (DW) test is the most widely used test for autocorrelation in regression model. But, this test has certain limitations, such as:

- Critical value depends on X-matrix, which leads to "indeterminate range".
- Only applicable for first order autocorrelation,
- Not suitable for dynamic model.

Durbin-Watson (DW) test is not applicable for small samples. In a case of small sample, DW test cannot be applied and in such a situation bootstrap procedure may be a solution to this problem. In this paper, we propose a bootstrapped DW-type test and compare the power properties by Monte-Carlo simulation.

The plan of this paper is as follows: In next section, discuss the model and hypothesis for testing first order autocorrelation. Third section discusses the techniques of bootstrapped DW statistic. In the section four, discuss the Monte-Carlo simulation study. Simulation study is carried out in section five.

MATERIALS AND METHODS: MODEL AND HYPOTHESIS

The model

Let us, consider the following regression model $Y = \beta X + u$ (1)

Where, Y and u are $(n \times 1)$, X is (n×k), β is $(k \times 1)$ and n denotes the number of observations. The disturbance vector, u is normally distributed with $E(u) = 0$ and $E(uu') \overset{.}{=} \Omega$, Ω is positive definite. We assume that the disturbance term follows a stationary AR(1) process of the form

$$u_t = \rho u_{t-1} + \epsilon_t, \text{ where, } \epsilon_t \sim NID(0, \sigma^2) \qquad (2)$$

Hypothesis

In order to test the autocorrelation in linear regression model as defined in (1) with disturbance term u, which follows an AR (1) process of the form (2), the hypothesis are as follows:

$$H_0: \rho = 0 \text{ against } H_a: \rho > 0$$

BOOTSTRAP DW (BDW) TESTS FOR AUTOCORRELATION

Many research work have been conducted considering bootstrapped based solution of the problem of autocorrelation. (Jeong and Chung (2001), MacKinon, G. J. (2005)). Davidson and Mackinnon (2002) applied a recursive bootstrap procedure to test of autocorrelation in the presence of lagged dependent variables. Their simulation results show that the bootstrap would be a useful tool for autocorrelation tests. The empirical sizes of bootstrap tests are acceptable, if not perfect. The power of a 'parametric' bootstrap test is closed to the nearest one except in some abnormal cases with a small number of observations. (Abrahamse, A. P. J. and Koerts (1969)).

In this study suggest a bootstrap test for autocorrelation, is called 'Bootstrapped DW(BDW) test'. Firstly, BDW test uses the critical values from the empirical bootstrap distribution and eliminates the indeterminate range of the DW test. Elimination of the inconclusive range expected to improve the power of the DW test. Secondly, the BDW test is robust to distributional assumptions. The (d_L, d_U) adjustment as well as the other approximated test procedures based on the normality assumption. Bootstrap utilizes the information embedded in a given finite sample; bootstrapping is expected to provide better finite sample properties then the asymptotic testing procedure.

It's not a question how much accurate the bootstrap DW statistics. One decide how to resample the data, how to construct the empirical distribution, whether to construct the empirical distribution under the null hypothesis or under the alternative hypothesis (i.e. at a given sample), etc. We apply the following recursive bootstrap procedure to the DW statistic.

i.   Estimate $\hat{\beta}$ of equation (1) by OLS and compute $\hat{u}$.
ii.   Estimate $\hat{\rho}$ of equation (2) by OLS and compute $\hat{\varepsilon}$.
iii.   Resample $\hat{\varepsilon}$ to construct a bootstrap residual vector ε^*.

iv. Recursively construct a bootstrap residual vector u^* by equation (2). At this stage, to impose the null hypothesis $H_0: \rho = 0$. As a result, u^* becomes identical to ε^*.

v. Using X and u^* create the 'fake' data by y^* by equation (1).

vi. Using X and y^* we recomputed the DW statistic d^*.

vii. Repeat step (III)-(VI) to construct the empirical distribution of d^*.

viii. We reject the null hypothesis when $d > d_j^*$. The bootstrap critical value c_α^* is the $(1 - \alpha)$ quintile of the d_j^*. This is simply a number $(1 - \alpha)*B$ in the list of d_j^*, sorted from smallest to largest.

MONTE-CARLO SIMULATION

To verify the finite sample performance of the bootstrap tests suggested in this paper, perform a Monte-Carlo study. We consider the three alternative tests for autocorrelation in the regression model: the original DW test, the BDW test and the Asymptotically DW (ADW) test. (see, for example, Gujarati (2003, pp471-472)).

The regression model (1) along with the first order error structure (2) is used in the simulation study. We consider a hypothesis of the form $H_0: \rho = 0$ against $H_a: \rho > 0$. In Monte-Carlo simulation, all β's are set to 1, and the X's are created by random picks from an uniform distribution over the range 0 to 1. The white noise error ϵ_t is a standard normal. The degree of autocorrelation is controlled by changing the value of ρ. In this study considered 10 different values of ρ: 0.0 ,0.1,…..0.9 and seven different sample sizes; 10,15, 20,30,50, 100, and 200. The number of bootstrap replication is set to 200, and the number of simulation is 1000. The normal size of the tests is set to 0.05.

RESULTS AND DISCUSSION

Table 1 and 2 present the Monte-Carlo study and the visual inspection of the results are shown in figure 1and 2. Table 1 and table 2 represent size and power of the three different types of DW test discussed in this paper for n=15 and n=20. Figure 1-2 present the power of three tests of result included in table 1-2. The vertical axis of the figures measures the power of three tests, and the horizontal axis represents the value of ρ.

From the results, it is observed several important implications. First, the original DW test has unacceptable small sample properties. The power of the DW test is not satisfactory when (n-k) is small. For example, when n=20 and k=10, the power of DW is 0.204, when n=10, and k=3, the power is 0.185 and when n=15, k=10, the power is as low as 0.075 even for $\rho = 0.9$.

But the BDW test shows better finite sample properties then DW test. For example, when n=20, and k=10, the power of BDW is 1.0, when n=10, and k=3, the power is 0.994 and when n=15, and k=10, the power is 0.813 for $\rho = 0.9$. These powers are always higher than the power of the DW and ADW.

It is evident from the tables and all figures that the power of BDW test is always higher than the usual DW test including Asymptotic DW test for different values of n and ρ considered in this study.

CONCLUSIONS

In this paper, a bootstrap Durbin Watson test has been proposed. Monte-Carlo study indicates that the bootstrap test show higher power than the existing DW and ADW tests for small sample.

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APPENDIX A

TABLE1: SIZE AND POWER OF THE ORIGINAL DW, BDW AND ASYMPTOTIC DW TESTS WHEN n=15, α=.05, ρ>0

k	Test	Size	Power								
	ρ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3	DW	0.049	0.063	0.079	0.101	0.169	0.201	0.318	0.418	0.504	0.611
	BDW	0.049	0.074	0.181	0.362	0.568	0.789	0.903	0.960	0.983	0.994
	ADW	0.05	0.074	0.127	0.186	0.259	0.309	0.416	0.553	0.629	0.723
5	DW	0.049	0.045	0.067	0.071	0.087	0.113	0.158	0.203	0.231	0.255
	BDW	0.049	0.058	0.099	0.180	0.294	0.430	0.565	0.661	0.730	0.810
	ADW	0.05	0.063	0.104	0.122	0.153	0.185	0.252	0.303	0.326	0.366
10	DW	0.049	0.041	0.050	0.066	0.072	0.067	0.055	0.084	0.072	0.075
	BDW	0.049	0.208	0.522	0.809	0.937	0.981	0.992	0.996	0.996	0.982
	ADW	0.05	0.056	0.078	0.092	0.105	0.102	0.106	0.117	0.108	0.106

TABLE2: SIZE AND POWER OF THE ORIGINAL DW, BDW AND ASYMPTOTIC DW TESTS WHEN n=20, α=.05, ρ>0

k	Test	Size				Power					
	ρ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3	DW	0.049	0.067	0.098	0.156	0.260	0.372	0.526	0.669	0.786	0.872
	BDW	0.049	0.125	0.354	0.646	0.864	0.964	0.992	0.998	0.999	0.999
	ADW	0.05	0.084	0.152	0.266	0.341	0.485	0.615	0.735	0.838	0.914
5	DW	0.049	0.049	0.079	0.126	0.171	0.255	0.343	0.464	0.540	0.648
	BDW	0.049	0.104	0.250	0.498	0.717	0.868	0.955	0.968	0.992	0.998
	ADW	0.05	0.071	0.132	0.212	0.270	0.367	0.450	0.572	0.667	0.775
10	DW	0.049	0.058	0.065	0.074	0.077	0.104	0.115	0.153	0.157	0.204
	BDW	0.049	0.487	0.689	0.802	0.899	0.942	0.978	1	1	1
	ADW	0.05	0.063	0.080	0.110	0.120	0.153	0.183	0.226	0.262	0.291

APPENDIX B
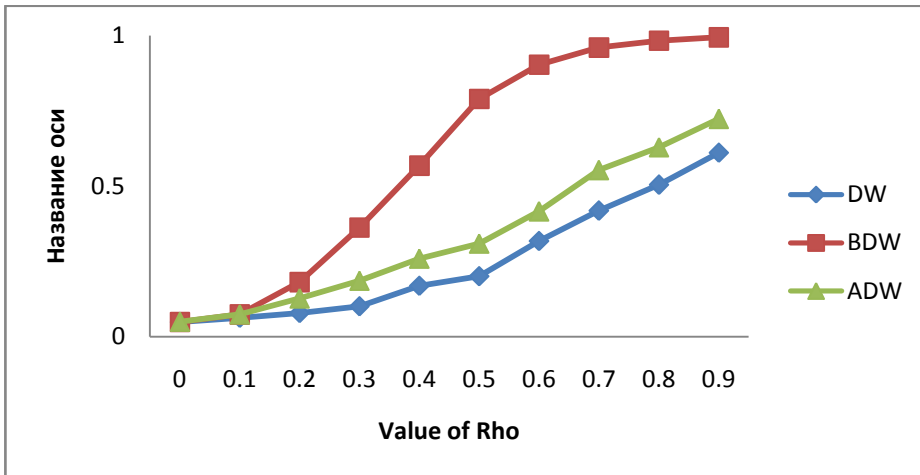

Figure 1: Illustration of the power curve of DW, and ADW test when n=15, k=3, with the proposed Bootstrapped based Durbin-Watson (BDW) test.
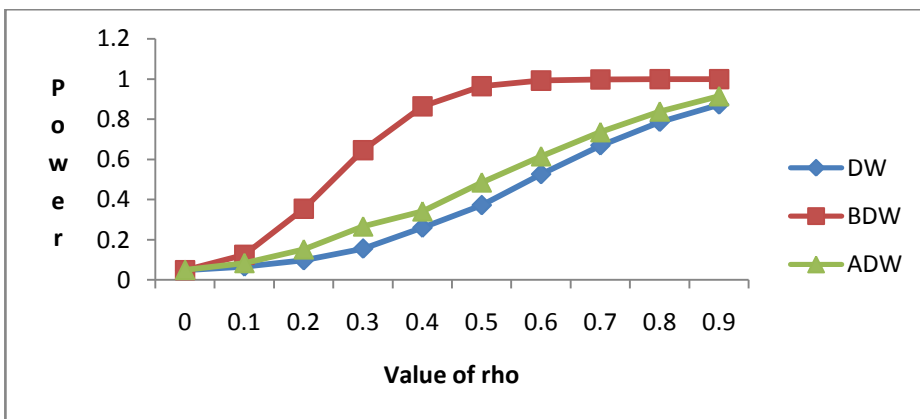

Figure 2: Illustration of the power curve of DW, and ADW test when n=15, k=3, with the proposed Bootstrapped based Durbin-Watson (BDW) test.

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