Asia Pacific Journal of Energy and Environment, Volume 1, No 1 (2014)

# Determination of energy gain time dependent in D+T mixture with calculating total energy deposited of deuteron beam in hot spot

**S. N. Hoseinimotlagh[1], M. Jahedi[2]**

[1]Sciences College, Physics Department, Pardis Islamic Azdad University of Shiraz, Pardis, Iran
[2]Department of Physics, Science and Research Branch, Islamic Azad University, Fars, Iran

## ABSTRACT

The fast ignition (FI) mechanism, in which a pellet containing the thermonuclear fuel is first compressed by a nanosecond laser pulse, and then irradiated by an intense "ignition" beam, initiated by a high power picosecond laser pulse, is one of the promising approaches to the realization of the inertial confinement fusion (ICF). If the ignition beam is composed of deuterons, an additional energy is delivered to the target, coming from fusion reactions of the beam-target type, directly initiated by particles from the ignition beam .In this work, we choose the D+T fuel and at first step we compute the average reactivity in terms of temperature for first time at second step we use the obtained results of step one and calculate the total deposited energy of deuteron beam inside the target fuel at available physical condition then in third step we introduced the dynamical balance equation of D+T mixture and solve these nonlinear differential coupled equations versus time .In forth step we compute the power density and energy gain under physical optimum conditions and at final step we concluded that maximum energy deposited in the target from D+T and D+D reaction are equal to to19269.39061 keV and 39198.58043 keV respectively.

**Key words:** Deuteronbeam, fast ignition, gain, dynamics

## INTRODUCTION

In 1975 A. W.Maschke suggested the use of relativistic heavy ion beams to ignite an inertially confined mass of thermonuclear fuel (Maschke, 1975). As in conventional inertial confinement fusion, the fuel was assumed to be precompressed by a factor of the order of 100 in order to minimize the energy needed for ignition. Maschke suggested that lasers might compress the fuel, by high velocity impact, or by ion beams other than the ignition beam. Maschke's "fast ignition" scheme was finally abandoned in favor of the more conventional approach to inertial fusion where the implosion supplies the energy for both compression and ignition. In 1994 an important paper by Tabak et al. (Tabak etal, 1994) rekindled interest in fast ignition ,this time using short-pulse lasers to provide the ignition temperature. Later, at the 1997 HIF Symposium, Tabak estimated requirements for heavy-ion-driven fast ignition (Tabak etal, 1997).This approach is now the subject of intense investigation in a number of countries. If ion beams could be made to deliver the energy density needed for ignition, they would have a number of distinct advantages. The reliability, durability, high repetition rate, and high driver efficiency are expected to be advantages of any accelerator driven inertial fusion system. In the case of fast ignition, there are some additional advantages. Moreover, for ions of the appropriate range, the beam energy can be deposited directly in the fuel, eliminating the inefficiency of converting laser light to electrons or ions that then deposit their energy in the fuel. Finally, because of the reduced requirements on illumination symmetry and stability, it may be possible to devise simple illumination schemes using direct drive or tightly coupled indirect drive, or use of single sided ion illumination for indirect drive fuel compression. This could simplify chamber design and, since direct drive and or tightly coupled indirect drive are efficient implosion methods, it could lead to lower driver energy and, as in the laser case, higher energy gain. We consider ion beam requirements for fast ignition in general, where a single short pulse of ions comes in from one direction onto one side of a pre-compressed D+T fuel mass, heating a portion of that fuel mass to conditions of ignition and propagating burn in a pulse shorter than the time for the heated region to expand significantly. The igniter beam (or beams) would be arranged to penetrate the target in such a way that the Bragg peak occurs at the usual target hot spot. Or one might increase the expansion time by tamping the ignition region. In fact Magelssen published a paper in 1984 (Magelssen, 1984) in which he presented calculations of a target driven by ion beams having two very different energies. The lower energy ions arrived first and imploded the target to a spherical configuration with a rather dense pusher or tamper surrounding the fuel. The higher energy beams were then focused onto the entire assembly, heating both the fuel and the pusher. The combination of the exploding pusher and the direct ion energy deposition heated the fuel to ignition.

In summary, the fast ignition (FI) mechanism, in which a pellet containing the thermonuclear fuel is first compressed by a nanosecond laser pulse, and then irradiated by an intense "ignition" beam, initiated by a high power picosecond laser pulse, is one of the promising approaches to the realization of the inertial confinement fusion (ICF). The ignition beam could consist of laser-accelerated electrons, protons, heavier ions, or could consist of the laser beam itself. It had been predicted that the FI mechanism would require much smaller overall laser energies to achieve ignition than the more conventional central hot spot approach, and that it could deliver a much higher fusion gain, due to peculiarities of the pressure and density distributions during the ignition. It is clear, however, that interactions of electrons and ions with plasma, and most importantly the energy deposition mechanisms are essentially different. Moreover, if the ignition beam is composed of deuterons, an additional energy is delivered to the target, coming from fusion reactions of the beam-target type, directly initiated by particles from the ignition beam (Sherlock etal, 2007). These and other effects had been of course taken into account in later works on this topic. In this work, we choose the D+T fuel and at first step we compute the average reactivity in terms of temperature for first time at second step we use the obtained results of step one and calculate the total deposited energy of deuteron beam inside the target fuel at available physical condition then in third of step we introduced the dynamical balance equation of D+T mixture and solve these nonlinear differential coupled equations by programming maple-15 versus time .in forth step we compute the power density and energy gain under physical optimum conditions and at final step we analyzed our obtained results.

## NEW CALCULATION OF AVERAGE REACTIVITY FOR D+T AND D+D REACTIONS

In order for two nuclei to fuse they must be brought close to each other such thatthe strong nuclear force dominates the repulsive Coulomb force. Since this Coulombforce preventing two nuclei from getting closer is an increasing function of the atomicnumber Z, the most promising fusion reactions are those involving Hydrogen and itsisotopes, namely Deuterium and Tritium. The most widely investigated reactions arethe so-called D+Dreaction and D+Treaction listed below.

The D+Dreaction:

$$D + D \rightarrow {}^{3}_{2}He(0.82MeV) + n(2.45MeV) \qquad \%50 \qquad (1)$$

$$D + D \rightarrow T(1.01MeV) + p(3.03MeV) \qquad \%50 \qquad (2)$$

The D+Treaction:

$$D + T \rightarrow {}^{4}_{2}He(3.5MeV) + n(14.1MeV) \qquad (3)$$

The rate of thermonuclear reaction is a function of the reaction cross section, temperature , and density. For a mixture of two species with densities $n_i$ and $n_j$ , the reaction rate is given by theexpression (Atzeni etal, 2002),

$$R_{ij} = \frac{n_i n_j}{1 + \delta_{ij}} < \sigma v > \qquad (4)$$

where $\sigma$ is the reaction cross section and $\delta_{ij}$ is the Kronecker symbol .If the fusion is "Thermonuclear", i.e. if the energy provided to the reacting particles is the result of heating the system to a temperature T, then we need to use the averaged $< \sigma v >$. Assuming a Maxwellian distribution, equation (4) becomes,

$$R_{ij} = \frac{n_i n_j}{1 + \delta_{ij}} \left[\frac{8}{\pi \mu}\right]^{3/2} \left[\frac{1}{kT}\right]^{\frac{3}{2}} \int_0^\infty \sigma(E) E \, exp\left(-\frac{E}{kT}\right) dE \qquad (5)$$

By comparing equation (4) and (5) we have

$$< \sigma v > = \left[\frac{8}{\pi \mu}\right]^{3/2} \left[\frac{1}{kT}\right]^{\frac{3}{2}} \int_0^\infty \sigma(E) E \, exp\left(-\frac{E}{kT}\right) dE \qquad (6)$$

Since the D+Treaction has the highest reaction rate at relatively low fuel temperature, it is the easiest fuel to assemble and burn. However, the presence of neutrons as a product (see D+T reaction) makes it unattractive for fusion reactors.The formula of fusion cross section for all these fusion reactions is given by (Xing, 2008):

$$\sigma(E_{lab}) = -16389 C_3 \left(1 + \frac{m_a}{m_b}\right)^2 \times \left[ m_a E_{lab} \left[ Exp\left( 31.40 \, Z_1 Z_2 \sqrt{\frac{m_a}{E_{lab}}} \right) - 1 \right] \left\{ (C_1 + C_2 E_{lab})^2 + \left( C_3 - \frac{2\pi}{\left[Exp\left(31.40 Z_1 Z_2 \sqrt{m_a / E_{lab}}\right) - 1\right]} \right)^2 \right\} \right]^{-1} \qquad (7)$$

with 3 adjustable parameters ($C_1$, $C_2$ and $C_3$) only. In (7), $m_a$ and $m_b$ are the mass number for the incident and target nucleus, respectively (e.g. $m_a$ = 2 for incident deuteron);$E_{lab}$ (deuteron energy in lab system) is in units of keV and $\sigma$is in units of barn. The numerical values of $C_1$, $C_2$ and $C_3$ for these reactions are listed in Table.1.

Table 1.Numerical values of $C_1$, $C_2$ and $C_3$ for reactionsD+Tand D+D(Xing, 2008).

| | D+T | D+D (p+T and n+$^3$He) |
|---|---|---|
| $C_1$ | −0.5405 | −60.2641 |
| $C_2$ | 0.005546 | 0.05066 |
| $C_3$ | −0.3909 | −54.9932 |

From this formula, we calculated and plotted the variations of fusion cross sections for these reactions in terms of$E_{lab}$ and our results are shown that in Figure.1. Also by comparing our calculated numerical values with available experimental results as are shown in (Xing, 2008), we concluded that this formula is very exact.

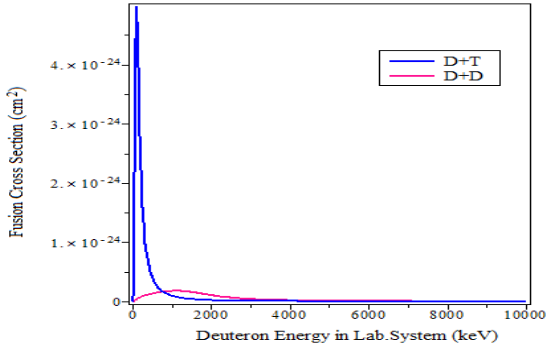



Figure1: Variations of fusion cross sections versus deuteron energy in lab system for different fusion reactions of D+Tand D+D

By using data in Table. 1 and inserting equation (7) into equation(6) and integrate it for these reactions we can obtain the numerical values of averaged reactivity parameter in terms of temperature and plotted it for D+T and D+D in the Figure.2.Note that $E_{lab} = \frac{m_a+m_b}{m_b}E$ ,where$E$ is energy in the center of mass frame. By observing Figs.1 and 2 we concluded that the cross section and averaged reactivity of D+T fusion reaction is greater than D+D, and $<\sigma v>_{D+T}$ *and* ,$<\sigma v>_{D+D}$ are strongly temperature dependent .From Fig.2, it will therefore be recognize that resonant temperature in which the probability for occurring fusion is maximized for both D+T and D+D fusion reactions are approximately 60and 300keV ,respectively.

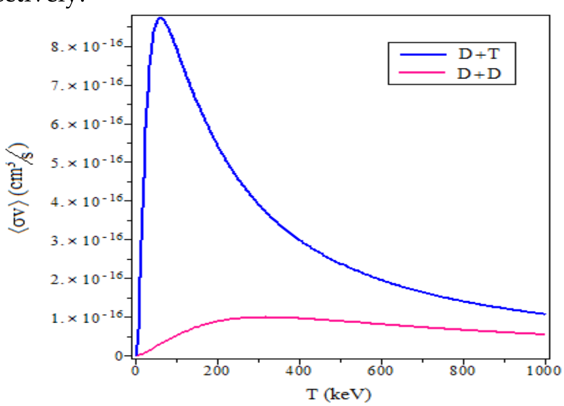



Figure2: Variations of averaged reactivity in terms of temperature for $<\sigma v>_{D+T}$,< $\sigma v$>$D+D$.

## GAIN REQUIREMENT

In order for fusion to be economically an attractive energy source it is necessary that the energy from thermonuclear reaction exceeds both the energy invested in achieving it and the losses due to radiation, reactor wall, conversion inefficiencies, etc. A simple estimate of the driver efficiency and target gain can be obtained using energy bookkeeping approach (Fig.3).

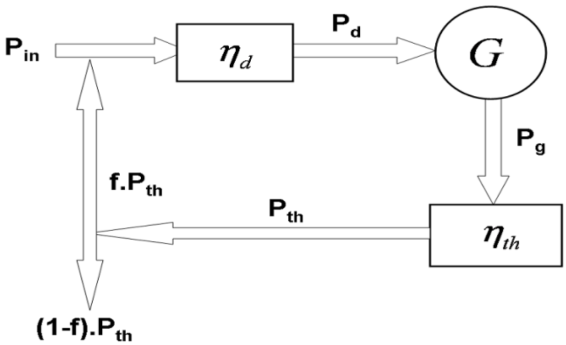



Figure 3: Power cycle.$P_{in}$, the input power (this could be the electrical powerfrom the outlet).$P_d$, the power at the output of the driver.$P_g$, the power fromthermonuclear fusion. $P_{th}$, the power after converting the thermal energy to electricalenergy.

Let $P_{in}$ be the electrical power that we intend to invest in achieving fusion and let $\eta_d$ be the efficiency of conversion of this power to laser light power $P_d$. If we define the target gain as the ratio of the power obtained from the fusion reaction to thedriver power, then

$$P_g = G\,P_d = G\,\eta_d P_{in} \qquad (8)$$

taking into account the conversion efficiency from thermal to electrical power, the available electrical power is given by,

$$P_{th} = \eta_{th}\,P_g = G\,\eta_{th}\,\eta_d P_{in} \qquad (9)$$

A fraction $fG\,\eta_{th}\,\eta_d P_{in}$ of this power is used to run the driver and the remainingfraction $(1-f)G\,\eta_{th}\,\eta_d P_{in}$ is send to the customers. The closer the factor $(1-f)$to unity, the more electricity is available to customers. This can be achieved by usingsmall values of$f$, however, $f$ has a lower bound determined by the equation,

$$P_{in} = f_{min}\,G\,\eta_{th}\,\eta_d P_{in} \rightarrow f_{min} = \frac{1}{G\,\eta_{th}\,\eta_d} \qquad (10)$$

Using a typical $\eta_{th}$ value of 40% and recycling less than 1/4 of the power to run thedriver, we get, $G\eta_d \geq 10$.Solid state lasers for example have an efficiency of 1-10% (Velarde etal) and to fulfill the available condition, gains of 100-1000 are needed.

قبلا بهره 1000 بود اما الان بهره در حدود 20 شده این جمله را نمی خواد حذف کرد؟

## DEUTERON BEAM AND DEPOSITED ENERGY IN PRE-COMPRESSED D+T FUEL

As we mentioned that in section 1, deuterons have been considered for fast ignition as well. By chenkov et al., considered an accelerated deuteron beam, but decided that deuterons would have too high an energy (7–8 MeV) to form the desired hot spot (Bychenkov etal, 2001). Deuterons would not only provide proven ballistic focusing, but also fuse with the target fuel (both D and T) as they slow down (Bathke etal, 1973), providing a “bonus” energy gain. Depending on the target plasma conditions, this added fusion gain can be a significant contribution (Shmatov, 2004). We must notice that the idea of bonus energy for first time is presented by Xiaoling Yang and et.al in low temperatures (Xiaoling, 2011), In this work we use of this idea, to compute the added energy released as the energetic deuterons interact with the target fuel ions in different range of temperatures ( 0-70keV) that is contained resonant temperature. This added energy increases total energy gain of the system. We use a modified energy multiplication factor $\varphi$ to estimate the bonus energy in terms of the added "hot spot" heating by beam-target fusion reaction for D+T (Bathke etal, 1973). The deuteron beam deposited energy and stopping range and time are also calculated for this reaction. Also we determine time dependent of energy gain in different temperatures.

The F value is the ratio between the fusion energy $E_f$ produced and the ion energy input $E_I$ to the plasma and for D+T fuel is given by (Bathke etal, 1973):

$$F_{D+T} = n_T \frac{\int_{E_{th}}^{E_I} S(E)dE}{E_I} \tag{11}$$

where $E_I$ and $E_{th}$ are, respectively, the average initial energy and the asymptotic thermalized energy of the injected single ion for each reaction[18,21,22] and

$$S(E) \equiv \sum_k K_k [< \sigma v(E) >_b]_{Ik} (E_f)_{Ik} / \left(\frac{dE}{dt}\right) \tag{12}$$

Such that for this reaction we have:

$$\frac{1}{n_T}\left(\frac{dE}{dt}\right) = -\frac{Z_I^2 e^4 m_e^{1/2} E \ln \Lambda_{D+T}}{3\pi(2\pi)^{1/2}\varepsilon_0^2 m_I (kT_e)^{3/2}} \times \left[1 + \frac{3\sqrt{\pi} m_I^{3/2} (kT_e)^{3/2}}{4 m_k m_e^{1/2} E^{3/2}}\right] \tag{13}$$

where$m_e$ is the mass of electron and $m_I$ is the mass of the injected ion, both of which are in atomic mass unit (amu).$< \sigma V >_{Ik}$ is the fusion reactivity for the injected ion $I$ of species $k$ having atomic fraction $K_k$ in the target, $(E_f)_{Ik}$is the corresponding energy released per fusion, and $T_e$ is the target electron temperature (Miley, 1976). By combining equations (11),(12) and (13) we can estimate $F_{D+T}$.

$\ln \Lambda_{D+T}$ is the Coulomb logarithm for D+T fusion reaction that is given by (Xiaoling, 2011):

$$\ln \Lambda_{D+T} \approx 6.5 - \ln\left(Z_k \sqrt{\rho}/T_e^{3/2}\right) \tag{14}$$

Where $Z_k$is the atomic number of target fuel ions and$T_e$ here is in kilo-electron-volt, ρ is in $\frac{g}{cm^3}$.We plotted the two and three dimensional variations of $\ln \Lambda_{D+T}$ in terms of target density and temperature(see Figure 4).

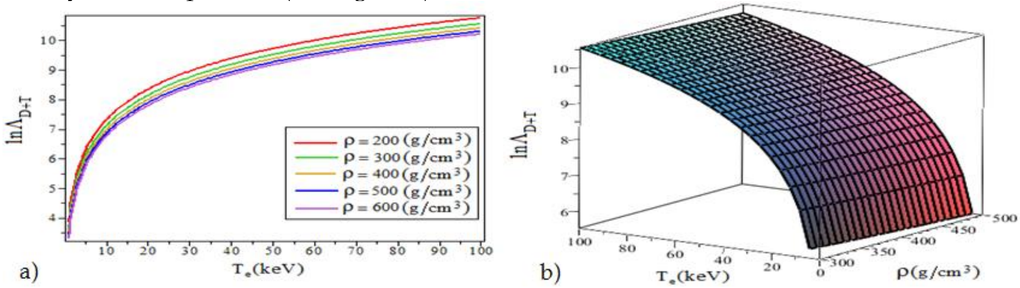



Figure 4: a) The two and b) three dimensional variations of$\ln \Lambda_{D+T}$ in terms of target density and temperature .

From this figure we see that by increasing temperature and reducing target density $\ln \Lambda_{D+T}$ is increased. The $(E_f)_{Ik}$in Eq. (12) gives the energy released in the fusion reaction carried by both fast neutrons and charged particles. However, for hot spot heating, the $\rho R$ of the hot spot is too low to significantly slow down the neutrons, so only charged particles contribute. Thus, for D+T reaction in the target(see reaction (3)) only the 20% of the fusion energy carried by the alphas is useful for heating while for the D+D reaction about 63% of the total is useful(see reactions (1) and (2) (Xiaoling, 2011).Therefore, to prevent confusion, we introduce a new factor $\varphi$ to represent the energy multiplication for the hot spot heating by the charged particles, and then $\varphi_{D+T} = 20\%\ F_{D+T}$ for D+T fusion and for occurring D+D fusion in D+T mixture $\varphi_{D+D} = 63\%\ F_{D+D}$.

In summary, the total energy that could be deposited into the target due to combined deuteron ion heating and beam-target fusion for D+T and D+D , respectively becomes:

$$\varepsilon_{D+T} = E_I(1 + \varphi_{D+T}) \qquad (15)$$

$$\varepsilon_{D+D} = E_I(1 + \varphi_{D+D}) \qquad (16)$$

so it is seen that the parameters $\varphi_{D+T}$ ,$\varphi_{D+D}$ play the role in the "bonus energy" for deuteron driven fast ignition in two reactions.To avoid confusion, please note that the $\varepsilon$ here is the total energy deposited by the ion beam plus any contribution from its beam-target fusion in the hot spot, but not the total input energy to the target that is often cited in energy studies and represents the total laser compression plus fast ignition energy delivered to the total target ,also the deuteron stopping range and stopping time can be calculated by following equations (Xiaoling, 2011):

$$R_S = \int_{E_{th}}^{E_I} v_D\, dE \Big/ \left(\frac{dE}{dt}\right) \qquad (17)$$

$$t_S = \int_{E_{th}}^{E_I} dE \Big/ \left(\frac{dE}{dt}\right) \qquad (18)$$

Where, $\left(\frac{dE}{dt}\right)$is calculated from equation (13) for D+T, the deuteron velocity is $v_D = \sqrt{\frac{2E}{m_D}}$.

Forcalculating the total energy deposited into the target of D+T at first step we substitute equation (14) into equation (13),then at second step the obtained result is substituted into equation (12) and at third step the result of second step are inserted into equation (11) and we compute $F_{D+T}$ and $F_{D+D}$ for D+T reaction,at forth step we use of these parameters for determination of $\varphi_{D+T}$and$\varphi_{D+D}$ at final step the obtained results from forth step inserted into equations (15) and (16) thus we can obtain the numerical values of $\varepsilon_{D+T}$and$\varepsilon_{D+D}$ in D+T mixture for $300 \le \rho(\frac{g}{cm^3}) \le 500, 0 \le T_e(keV) \le 70$ and deuteron energyE, with range of $0 \le E(MeV) \le 10$. Also under these conditions we can calculate the deuteron stopping range and stopping time by using equations(17) and (18).Figure.5 show the results of our calculations of$\varphi_{D+T}$ , $\varphi_{D+D}$ ,$\varepsilon_{D+T}$ and$\varepsilon_{D+D}$ in the case of $\rho = 500(g/cm^3)$ and in Table.2computed numerical values of maximum of$\varphi_{D+T}$, $\varphi_{D+D}$ , $\varepsilon_{D+T}$ and $\varepsilon_{D+D}$ are given for studyingD+T reaction at different temperatures and target density$300 \le \rho(g/cm^3) \le 500$.

We see clearly that from Fig.5, the changes of$\varphi_{D+T}$ , $\varphi_{D+D}$ ,$\varepsilon_{D+T}$ and$\varepsilon_{D+D}$ by increasing temperature is such that we have the maximum value of them in resonance temperature of 60 keV. We must be notice that the temperature 60 keV is a resonant temperature for D+T reaction such that in this temperature we have maximum probability for occurring fusion reaction. Also from Table.2 we find that the maximum total deposited energy of $\varepsilon_{D+T}$ and$\varepsilon_{D+D}$ inside hot spot and also the maximum of $\varphi_{D+T}$ and$\varphi_{D+D}$ by increasing target density from 300 to $500(g/cm^3)$are raised very slowly.The total energy deposited inside hot spot ($\varepsilon_{D+T}$ and$\varepsilon_{D+D}$ ) is alsofunction of deuteron energy beam ($E_I$)such that by increasing energy from 1 to 10000keVincreased.Fig.5 show that the values of multiplication factors$\varphi_{D+T}$ , $\varphi_{D+D}$ increased by increasing deuteron energy(please note that our calculations show that in temperature lower than 5keV at first by raising the deuteron energy $\varphi_{D+T}$ and$\varphi_{D+D}$ are increased and then slowly decreased).In all temperatures $\varepsilon_{D+D}$ is higher than $\varepsilon_{D+T}$.From Fig.6 we see that by increasing temperature stopping time($t_{S_{D+T}}$) is increased because by raising temperature particle energy is increased then the stopping time become large. Since that by increasing temperature stopping range is increased then particle can move more distance (see Fig.7). The effective parameter that decreased stopping time and stopping range, is $n_T$. Our calculations show that by changing $n_T$ from $10^{22}$to$10^{24}cm^{-3}$,$t_{S_{D+T}}$ and $R_{S_{D+T}}$ are decreased by the order of 10 and 100. By increasing target density from 300 to 500$(g/cm^3)R_{S_{D+T}}$ and $t_{S_{D+T}}$ are increased, but this changes for $t_{S_{D+T}}$ is not very sensitive.

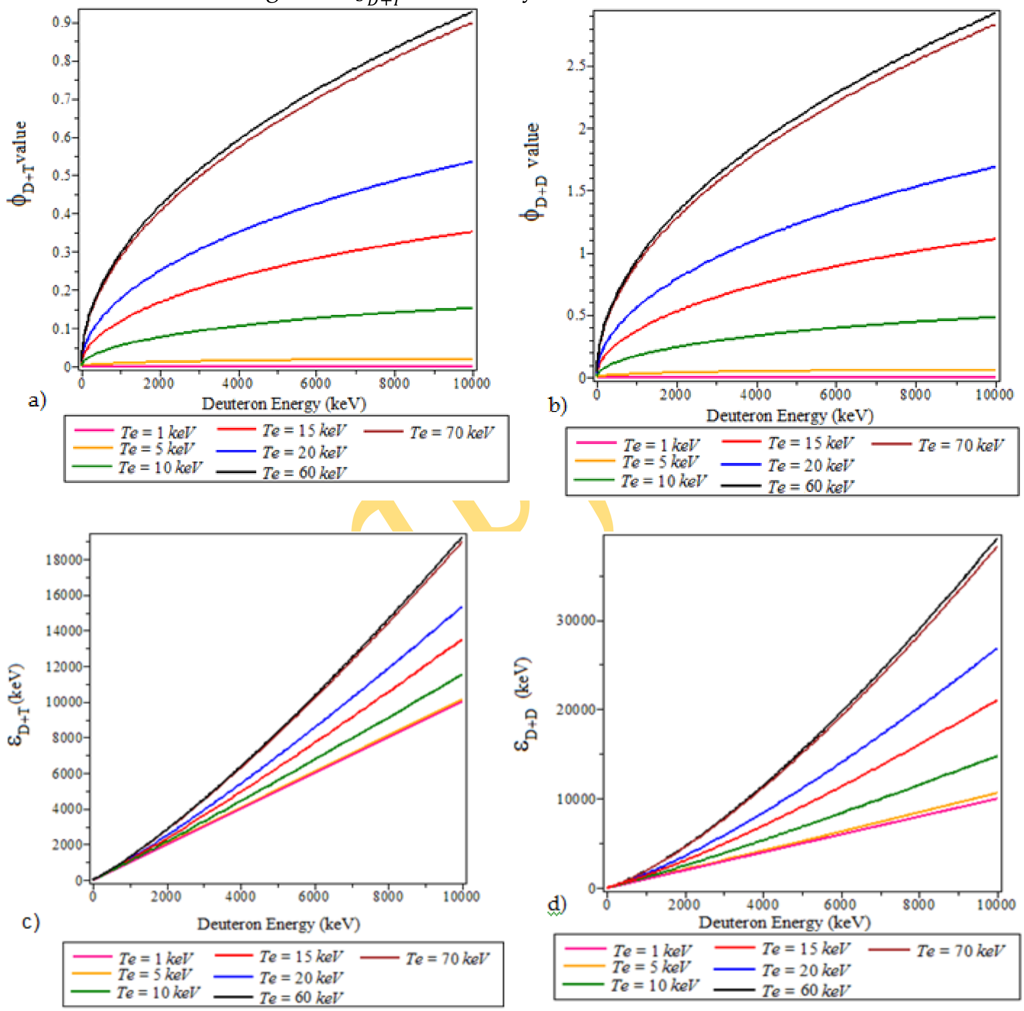
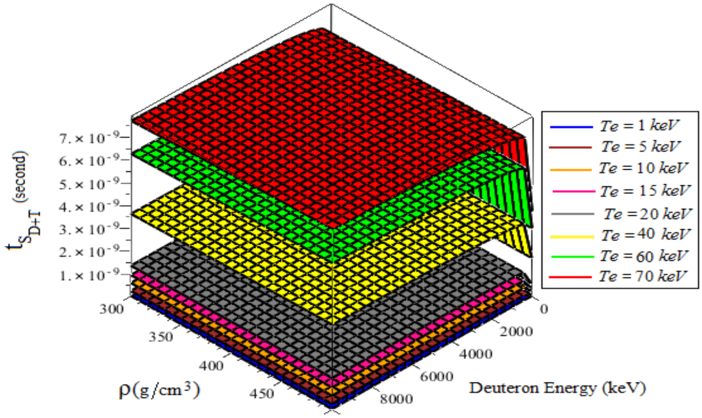


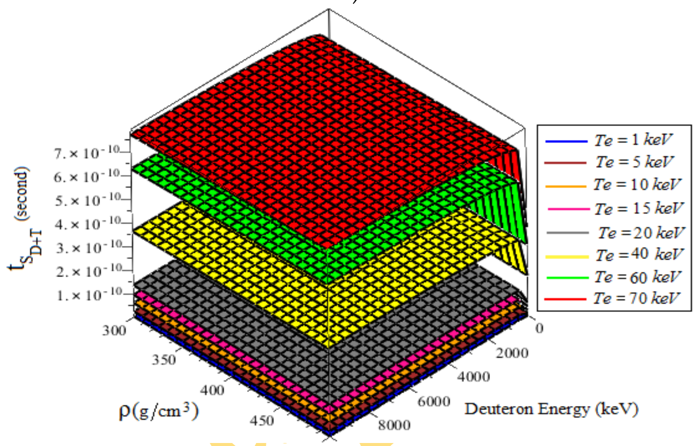

Figure 5: The two dimensional variation of a) $\varphi_{D+T}$ b)$\varphi_{D+D}$ c) $\varepsilon_{D+T}$ d) $\varepsilon_{D+D}$ in terms of deuteron energy in different temperature in D+T mixture for

$$\rho = 500\left(\frac{g}{cm^3}\right)$$

a)




b)




c)

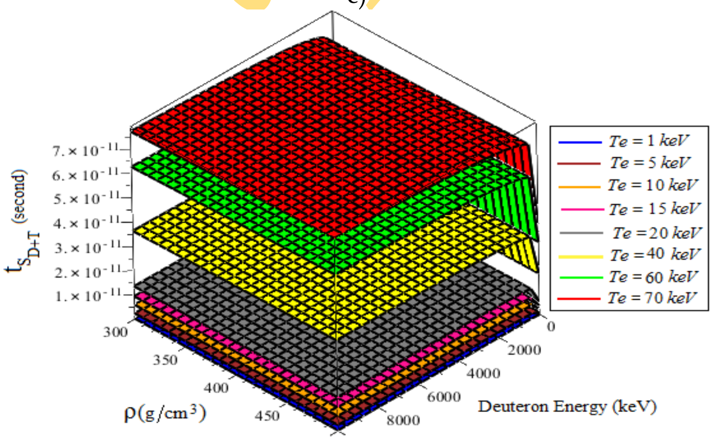



Figure 6: The three dimensional variations of stopping time versus target density and deuteron energy in different temperatures in D+T mixture a) $n_T = 10^{22}(cm^{-3})$ , b) $n_T = 10^{23}(cm^{-3})$ , c) $n_T = 10^{24}(cm^{-3})$.

a)

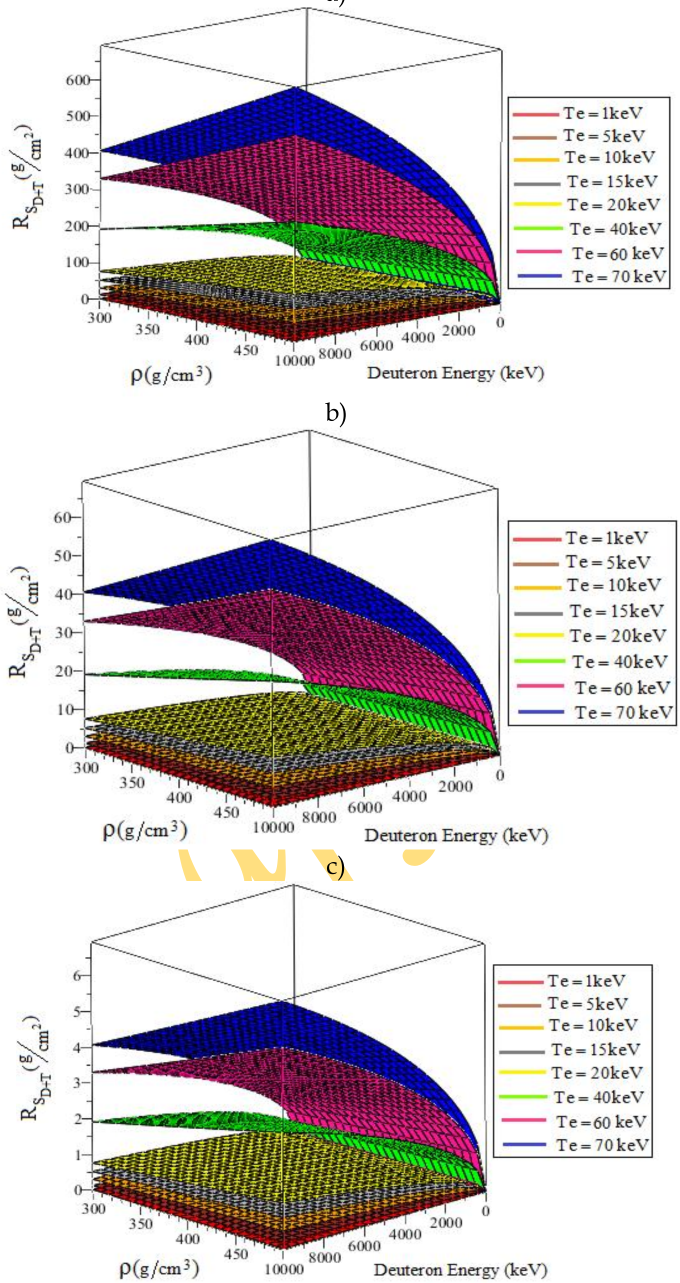



Figure 7: The three dimensional variations of stopping range versus target density and deuteron energy in different temperatures in D+T mixture a)$n_T = 10^{22}(cm^{-3})$ , b)$n_T = 10^{23}(cm^{-3})$ , c)$n_T = 10^{24}(cm^{-3})$.

Table.2: Maximum numerical values of total energy deposited and multiplication factors in D+T mixture at different temperature for $300 \le \rho(\frac{g}{cm^3}) \le 500$.

| D+T | | | | | |
|---|---|---|---|---|---|
| $\rho(g/cm^3)$ | $T_e$(keV) | $\varepsilon_{D+T_{max}}$ (keV) | $\varepsilon_{D+D_{max}}$ (keV) | $\varphi_{D+T_{max}}$ | $\varphi_{D+D_{max}}$ |
| 300 | 10 | 11463.92692 | 14611.36978 | 0.146392691 | 0.461136978 |
| 300 | 20 | 15185.35591 | 26333.87112 | 0.518535591 | 1.633387112 |
| 300 | 60 | 19020.60063 | 38414.89199 | 0.902060063 | 2.841489199 |
| 300 | 70 | 18763.52452 | 37605.10224 | 0.876352452 | 2.760510224 |
| 400 | 10 | 11494.18969 | 14706.69751 | 0.149418969 | 0.470669751 |
| 400 | 20 | 15278.61416 | 26627.63459 | 0.527861416 | 1.662763459 |
| 400 | 60 | 19159.04394 | 38850.98842 | 0.915904394 | 2.885098842 |
| 400 | 70 | 18891.14962 | 38007.12129 | 0.889114962 | 2.800712129 |
| 500 | 10 | 11518.539 | 14783.39786 | 0.151853900 | 0.478339786 |
| 500 | 20 | 15353.29364 | 26862.87495 | 0.535329363 | 1.686287495 |
| 500 | 60 | 19269.39061 | 39198.58043 | 0.926939061 | 2.919858043 |
| 500 | 70 | 18992.73249 | 38327.10736 | 0.899273249 | 2.832710736 |

## BALANCE EQUATIONS OF DEUTERIUM-TRITIUM MIXTURE

The following system of equations is used to describe the temporal evolution of plasma parameters averaged over the volume (the density of deuterium ions $n_D$ density of tritium ions $n_T$, density of thermal alpha-particles $n_\alpha$, plasma energy $E$, for D+T nuclear fusion reaction :

$$\frac{dn_D}{dt} = -\frac{n_D}{\tau_p} - n_D n_T \langle\sigma v\rangle_{D+T} + S_D \quad (19)$$

$$\frac{dn_T}{dt} = -\frac{n_T}{\tau_p} - n_D n_T \langle\sigma v\rangle_{D+T} + S_T \quad (20)$$

$$\frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_\alpha} + n_D n_T \langle\sigma v\rangle_{D+T} \quad (21)$$

The energy balance equationis given by

$$\frac{dE}{dt} = -\frac{E}{\tau_E} + Q_\alpha n_D n_T \langle\sigma v\rangle_{D+T} - P_{rad} \quad (22)$$

$S_D$and $S_T$are the source terms which give us the fuel rates; $\tau_\alpha$, $\tau_p$, and $\tau_E$are the lifetimes of thermal alpha-particles, deuterium and tritium, and the energy confinement time, respectively, also the energy of the alpha particles are: $Q_\alpha = 3.52 MeV$. The radiation loss $P_{rad}$ is given by:

$$P_{rad} = P_{brem} = A_b Z_{eff} n_e^2 \sqrt{T} \; for \; D+T \quad (23)$$

Where $A_b = 4.85 \times 10^{-37} \frac{Wm^3}{\sqrt{KeV}}$is bremsstrahlung radiation coefficient. No explicit evolution equation is provided for the electron density $n_e$ since we can obtain it from the neutrality condition $n_e = n_D + n_T + 2n_\alpha$ , whereas the effective atomic number, the total density and the energy are written as

$$Z_{eff} = \frac{\sum_i n_i Z_i^2}{n_e} = \frac{n_D + n_T + 4n_\alpha}{n_e} \quad (24)$$

where , $Z_i$ is the atomic number of the different ions. The fusion energy gain is defined as : $G(t) = \frac{E_f(t)}{E_{driver}}$ where $E_f(t)$ is equal to the energy due to the number of occurred fusion reactions in target in terms of time and $E_{driver}$ is the required energy for triggering fusion reactions in hot spot and is equal to 4MJ (Pfalzner, 2006). Also the fusion power density for D+T reaction is given by $P_{D+T} = n_D(t)n_T(t) < \sigma v >_{D+T} Q_{D+T}$ where $Q_{D+T} = 17.6MeV$ .

We solve time dependent nonlinear coupled differential equations (19) to (22) with the use of computers (programming, Maple-15) under available physical conditions. Our computational obtained results are given , in Figs.8 to 10 and Table.3.

From Figs.8 to 10 we see clearly that, by increasing temperature from 1 keV to 70 keV the variations of deuterium and tritium density in terms of time ($n_D(t), n_T(t)$) are decreased since that by increasing time the consumption rate of $n_D(t)$ and $n_T(t)$ are increased but the changes of $n_D(t)$ and $n_T(t)$ , in all temperature are similar and as we see in Figs.8 to 10 they coincide each other. Thus, by increasing temperature 1 keV to 70 keV the variations of alpha density ($n_\alpha(t)$) versus time at first by increasing time is increased and then decreased while the production rate of fusion plasma energy ($E_f(t)$) is increased until in resonant temperature 60 keV is maximized because in this temperature the highest number of D+T fusion reaction is occurred. Also, our calculations show that by increasing the injection rate of deuterium and tritium($S_D$ $and$ $S_T$) from $10^{22}$ to $10^{24} cm^{-3}$ the rate of variations of $n_D(t)$ and $n_T(t)$ in terms of time are increased while $n_\alpha(t)$ and $E_f(t)$ are increased. Therefore for having optimum value of power density and fusion energy gain by using the above discussions we choose the injection rate of deuterium and tritium and resonant temperature respectively equal to $10^{24} cm^{-3}$ and $T_e$ =60keV(see Table.3).

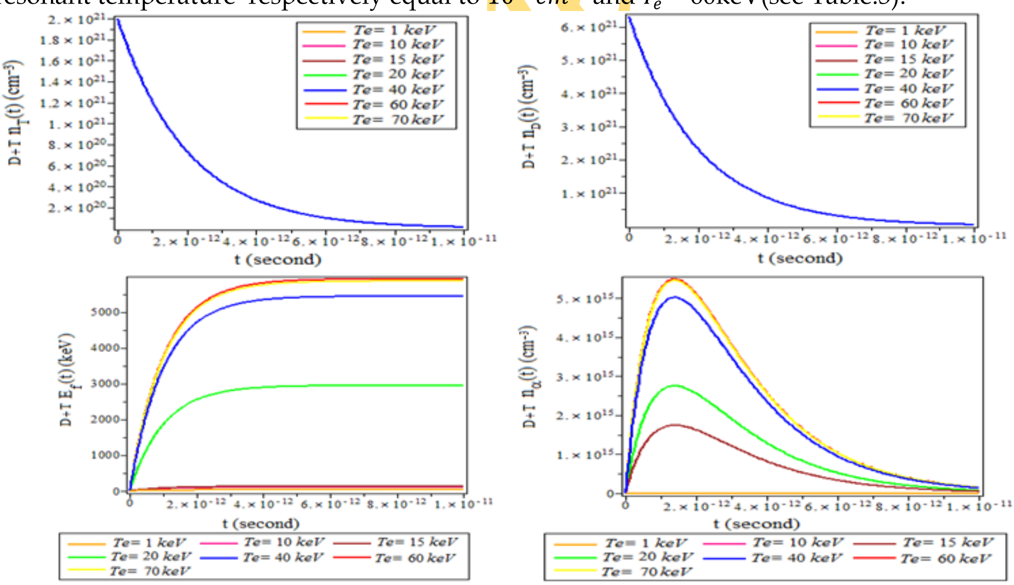



Figure 8: The two dimensional variations of deuterium , tritium and alpha particles densities and plasma energy in terms of time in different temperatures for D+T mixture under choosing $S_T = 0.20 \times 10^{22} (cm^{-3})$ and $S_D = 0.63 \times 10^{22} (cm^{-3})$.

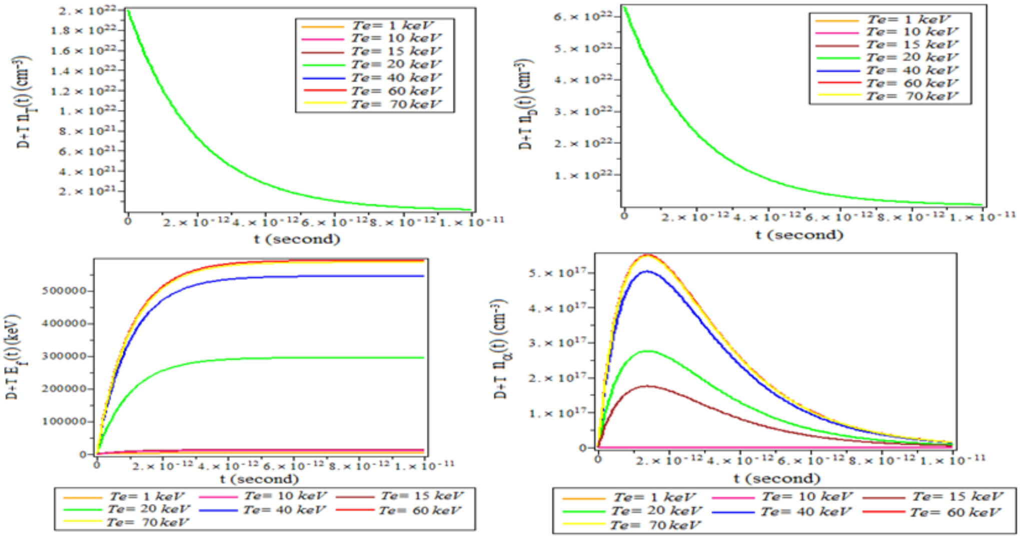



Figure 9: The two dimensional variations of deuterium , tritium and alpha particles densities and plasma energy in terms of time in different temperatures for D+T mixture under choosing $S_T = 0.20 \times 10^{23} (cm^{-3})$ and $S_D = 0.63 \times 10^{23} (cm^{-3})$.

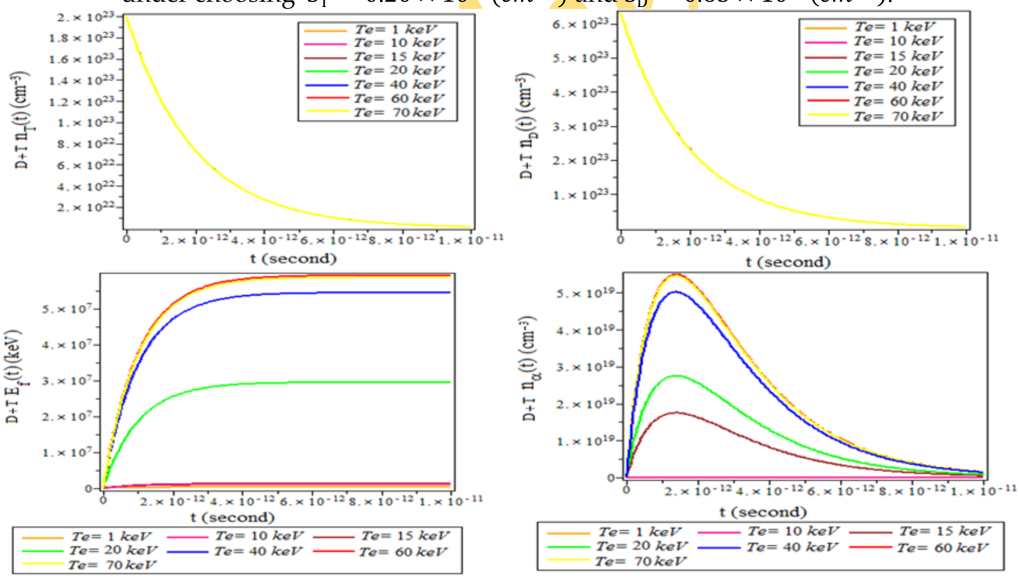



Figure 10: The two dimensional variations of deuterium, tritium and alpha particles densities and plasma energy in terms of time in different temperatures for D+T mixture under choosing $S_T = 0.20 \times 10^{24} (cm^{-3})$ and $S_D = 0.63 \times 10^{24} (cm^{-3})$.

Table.3: Time dependent numerical values of fusion power density and target energy gain.

| D+T<br>$S_D = 0.63 \times 10^{24}(cm^{-3})$ , $S_T = 0.20 \times 10^{24}(cm^{-3})$ | | | |
|---|---|---|---|
| $T_e$ (keV) | t (s) | $P_{D+T}(t)(\frac{W}{cm^3})$ | $G_{D+T}(t)$ |
| 20 | $10^{-20}$ | 1549.2E17 | 0.11802E-22 |
| 20 | $10^{-11}$ | 702.75E13 | 1.1798E-15 |
| 20 | 60 | 619.61E-6 | 0.572432E-7 |
| 20 | 110 | 619.61E-6 | 0.99075E1 |
| 40 | $10^{-20}$ | 2827.6E17 | 0.21788E-22 |
| 40 | $10^{-11}$ | 1282.1E13 | 2.1777E-15 |
| 40 | 60 | 1131.4E-6 | 1.0566E-6 |
| 40 | 110 | 1131.4E-6 | 1.8288E1 |
| 60 | $10^{-20}$ | 3092.4E17 | 0.23718E-22 |
| 60 | $10^{-11}$ | 1401.9E13 | 2.3705E-15 |
| 60 | 60 | 1236.9E-6 | 1.1501E-6 |
| 60 | 110 | 1236.9E-6 | 1.9906E1 |
| 70 | $10^{-20}$ | 3073.8E17 | 0.23485E-22 |
| 70 | $10^{-11}$ | 1393.5E13 | 2.3473E-15 |
| 70 | 60 | 1229.5E-6 | 1.1389E-6 |
| 70 | 110 | 1229.5E-6 | 1.9711E1 |



## Conclusion

From this paper, we conclude that the resonant temperature for D+T mixture is 60 keV and in this temperature we have maximum energy gain. Our computational results show that the numerical values of stopping time ($t_{s_{D+T}}$) are increased very strongly by raising temperature, while slowly increase by raising target density $\rho$. Another effective parameter on the value of stopping time is the deuteron energy such that by increasing this energy stopping time are increased. But by raising $n_T$in D+T reaction numerical values of stopping time strongly decreased. The important parameter that effect on the value of stopping range, is target density, by increasing target density $R_{s_{D+T}}$ is strongly increased. The multiplication factors and total deposited energies are time independent because in calculating these parameters $n_T$ is omitted. By observing the related figures and tables we concluded that the maximum variations of particle densities, plasma energy, power density and target energy gain are occurred in resonant temperature. From table.3 we have maximum gain equal to 19.906 at resonant temperature 60 keV under optimum conditions$S_D = 0.63 \times 10^{24}(cm^{-3})$ , $S_T = 0.20 \times 10^{24}(cm^{-3})$ ,t=110s.Finally, we concluded that the maximum energy deposited in the target from D+T and D+D reaction are equal to 19269.39061keVand 39198.58043keVrespectively. So, the deposited energy can reduce laser driver energy.

## References

A. Maksimchuk, S. Gu, K. Flippo, D. Umstadter, and V. Y. Bychenkov,Phys. Rev. Lett. 84, 4108 (2000).

A. W. Maschke, Proceedings of the (1975) Particle Accelerator Conference, page 1875,IEEE report NS-22, June (1975)

C. Bathke, H. Towner, and G. H. Miley, Trans. Am. Nucl. Soc. 17,41 (1973).
D. J. Rose and M. Clark, Jr., Plasmas and Controlled Fusion MIT Press, Cambridge, MA(1965).
G. H. Miley, Fusion Energy Conversion American Nuclear Society, Hinsdale, IL(1976).
G. Velarde, Y. Ronen, and J. Martinez-Val. Nuclear fusion by Inertial Confinement: A comprehensive treatise.
H. Schwoerer, S. Pfotenhauer, O. Jackel, K. U. Amthor, Ziegler, R. Sauerbrey, K. W. D. Ledingham, and T. Esirkepov ,Nature London439,445 (2006)
Logan, B. Grant Bangerter, Roger O.Callahan, Debra A.Tabak, Max Roth, Markus Perkins, L. John Caporaso, George ,Lawrence Berkeley National Laboratory (2005).
M Temporal J Honrubia, S Atzeni. Phys. Plas.9,3102(2002)
M. L. Shmatov, J. Br. Interplanet. Soc. 57,362(2004).
M. L. Shmatov, J. Br. Interplanet. Soc. 60,180 (2007).
M. Sherlock et al., Phys. Rev. Lett. 99, 255003 (2007).
M. Tabak, et al., Phys. Plasmas 1, 1626 (1994)
M. Tabak, et. al. Int. HIF Symposium (Heidelberg) paper ( 1997)
N. Naumova, T. Schlegel, V. T. Tikhonchuk, C. Labaune, I. V. Sokolov,and G. Mourou, Phys. Rev. Lett. 102, 025002 (2009).
S. Atzeni et al., Nucl. Fusion 49, 055008 (2009).
S. Atzeni, M. Temporal, and J. Honrubia, Nucl. Fusion 42, L1 (2002).
S. Pfalzner, "An Introduction to Inertial Confnement Fusion", Published by CRC Press Taylor & Francis Group(2006).
T. C. Magelssen, "Targets Driven by Dual-Energy Heavy Ions", Nuclear Fusion 24, 1527(1984).
V. Bychenkov, W. Rozmus, A. Maksimchuk, D. Umstadter, and C.Capjack, Plasma Phys. Rep. 27, 1017 (2001).
V. T. Tikhonchuk, T. Schlegel, C. Regan, M. Temporal, J.-L. Feugeas, P.Nicolaï, and X. Ribeyre, Nucl. Fusion 50, 045003 (2010).
Xiaoling Yang, George H. Miley, Kirk A. Flippo, and Heinrich Hora,PHYSICS OF PLASMAS 18, 032703 (2011)
Xing Z. Li, Qing M. Wei and Bin Liu, Nucl. Fusion 48 ,125003 ,5pp (2008).